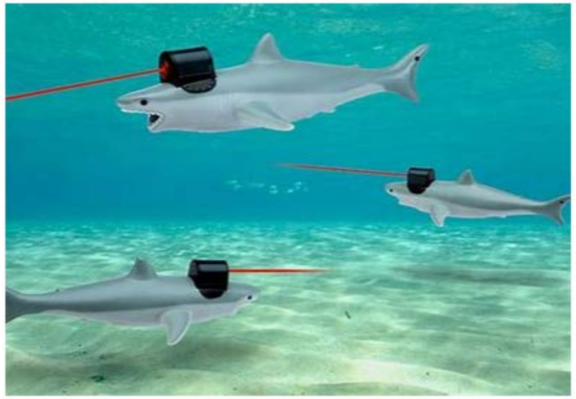
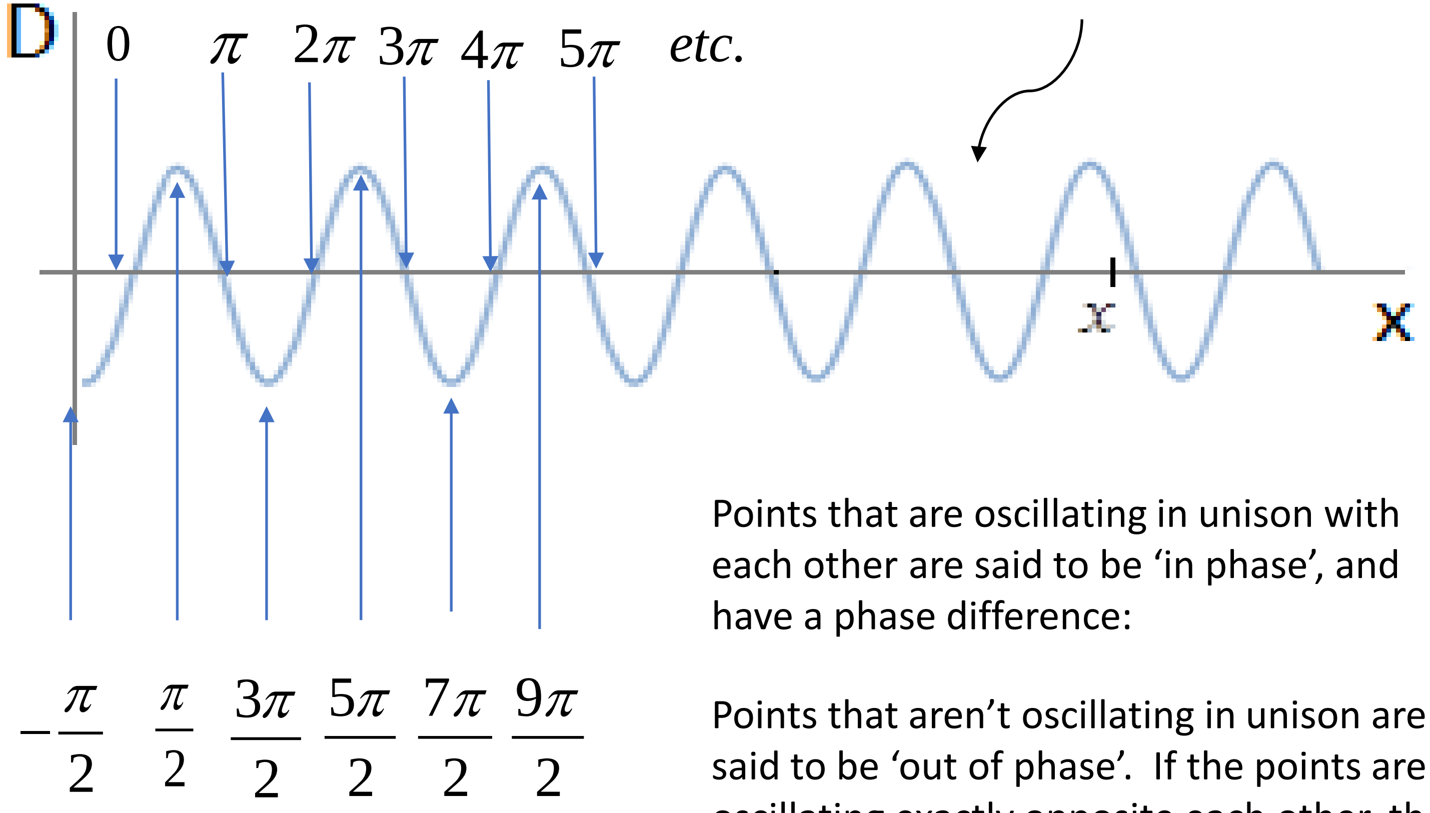
D. Resonance

Now we'll consider interference between these transmitting/reflecting waves and the patterns it can produce. In particular we'll find that under appropriate conditions, the waves will superimpose/interfere 'constructively' which will cause the wave to amplify prodigiously. This would manifest as a very 'loud' wave if it's sound, or a very 'bright' wave if it's light. And conversely we'll also find that under analogous conditions, the waves will superimpose/interfere 'destructively' which will cause the wave to cancel itself out. This would manifest as 'silence' in the case of sound, or a 'darkness' if its light. First we'll consider interference between the *same* wave with its reflections. To do so in the level of detail and generality necessary to encompass all of the phenomena we'll discuss, we have to introduce the concept of **phase**.



D.1 Phase

$$D(x,t) = A\sin(kx - \omega t + \varphi_0)$$



The phase of a wave at point x at time t is just the argument of its sin function.

$$\varphi = kx - \omega t + \varphi_0$$

Points that are oscillating in unison with each other are said to be 'in phase', and have a phase difference:

$$\Delta\varphi = 2\pi m \qquad m = 0, \pm 1, \pm 2 \pm 3, \ldots$$

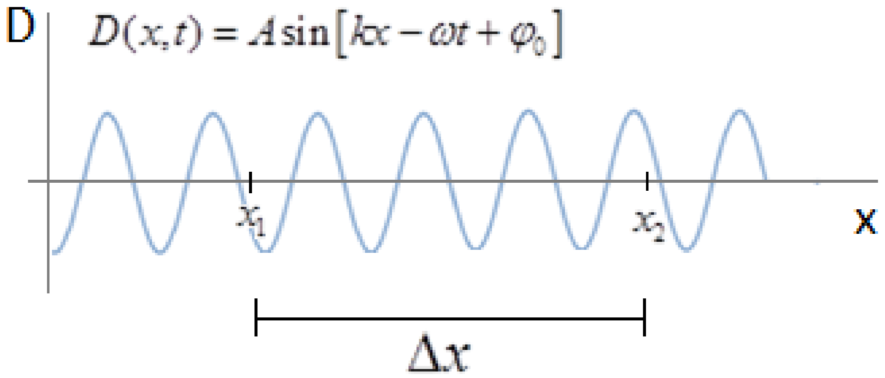
Points that aren't oscillating in unison are said to be 'out of phase'. If the points are oscillating exactly opposite each other, then they're said to be completely out of phase. *These* have a phase difference:

$$\Delta\varphi = 2\pi m_{1/2} \qquad m_{1/2} = \pm 0.5, \pm 1.5, \pm 2.5, \ldots$$

http://www.animations.physics.unsw.edu.au/jw/waves_phase_differences.htm

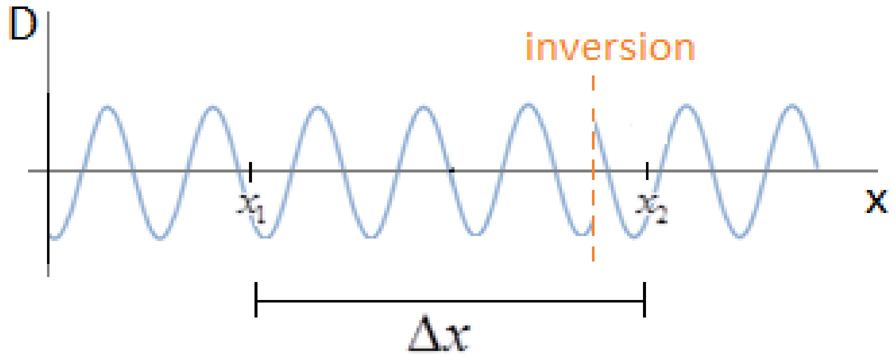
D.1 Phase

Particularly, we're interested in a formula for the difference in phase between two points on the wave, so that we can determine whether they're in phase or not. The formula is easy to establish….



$$\begin{aligned}\Delta\varphi &= \Delta(kx - \omega t + \varphi_0) \\ &= k\Delta x - \omega\Delta t + \Delta\varphi_0 \\ &= k\Delta x - 0 + 0 \\ &= k\Delta x\end{aligned}$$

But it could happen that the wave inverts at some point between those two points (as when it reflects off of harder medium), so then what?



Change in phase associated with inversion is π since,

$$-A\sin(kx - \omega t + \varphi_0) = A\sin(kx - \omega t + \varphi_0 + \pi)$$

So then, if wave inverts I times between x_1 and x_2, its change in phase between those points would be:

$$\Delta\varphi = k\Delta x + I\pi$$